

# Classification

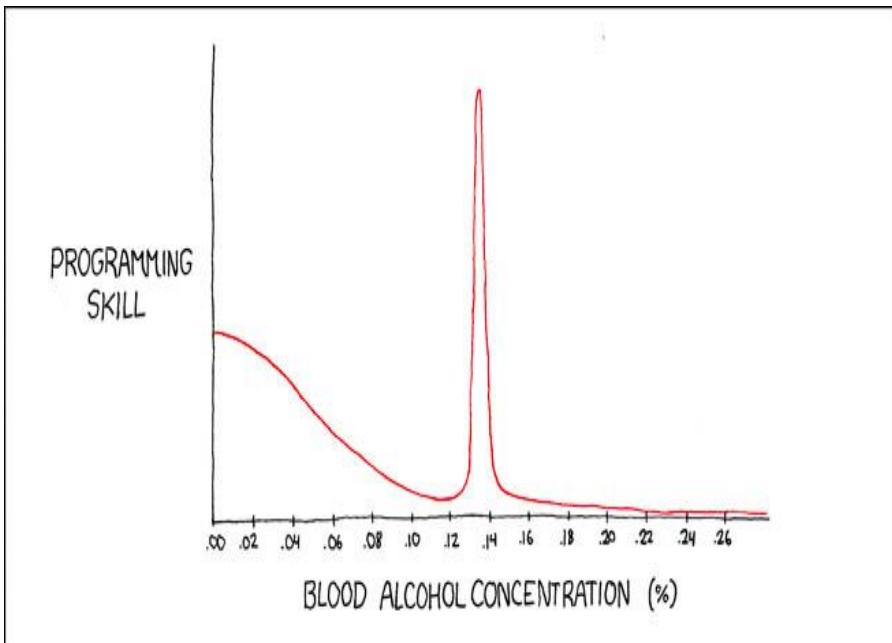


# Recap

We have learned regression models to predict numeric continuous variables.

- Linear Regression
- Logistic Regression
- Decision Tree

Ex: Predicting stock value, monthly temperature, etc.



# Intro to Classification

“What kind of species is this?”

“How would consumers rate this restaurant?”

“Which Hogwarts House do I belong to?”

“Am I going to pass this class?”



[Source](#)

# Conditional Probability

**Conditional Probability** - Probability of an event A *given* an event B. Normalize the probability of A and B by the probability of A. Written  $P(A|B)$ .

$$P(A | B) = \frac{P(B | A)P(A)}{P(B)}$$

**Independence** - Completely unrelated (very different from **uncorrelated**)

In terms of conditional probability, A and B are independent iff  $P(A|B) = P(A)$ .



# The Bayesian Classifier

- The ideal classifier: a theoretical classifier with the highest accuracy
- Picks the class with the highest conditional probability for each point
- Assumes conditional distribution is known
- Exists only in theory and does not exist in reality!
- A conceptual **Golden Standard**



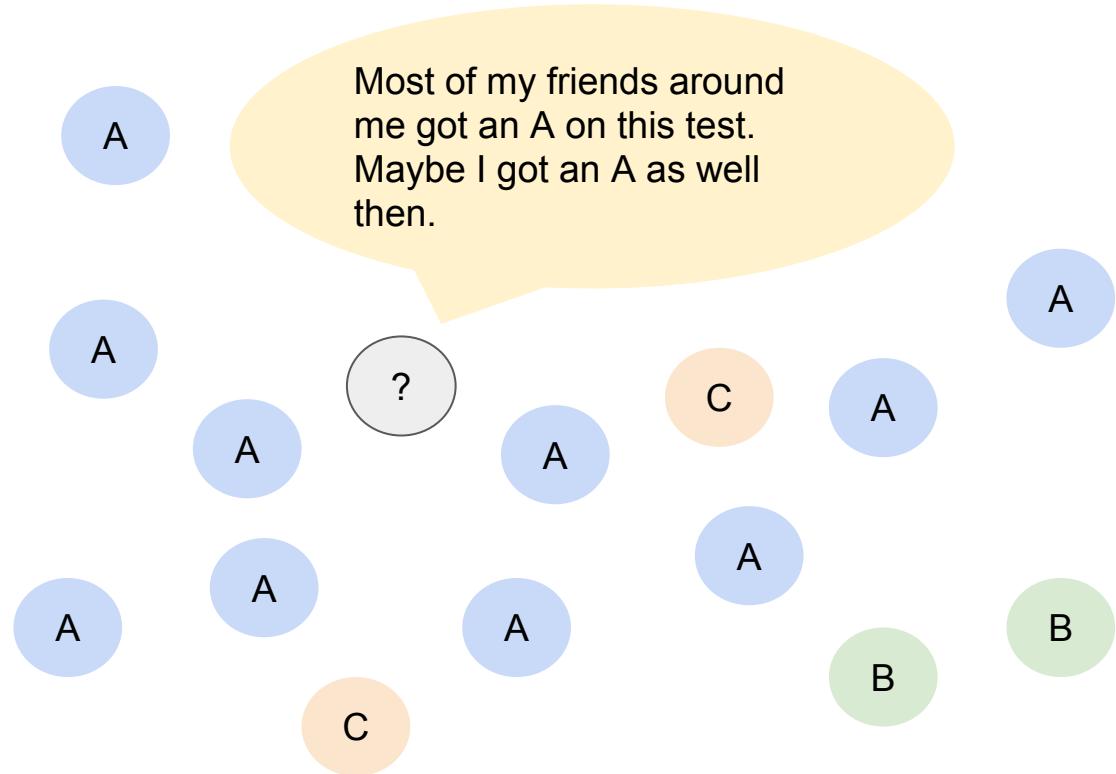
# Classifier 1: k-Nearest Neighbors (KNN)

Easy to Interpret

Fast calculation

No prior assumptions

Good for coarse analysis



# KNN

How does it work?

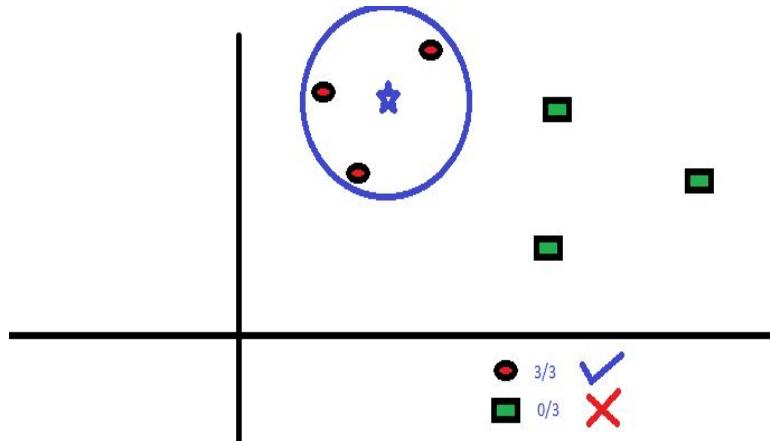
**Define** a  $k$  value (in this case  $k = 3$ )

**Pick** a point to predict (blue star)

**Count** the number of closest types

**Increase** the radius until the nearest type adds up to 3

**Predict** the blue star to be a red circle!



<https://www.analyticsvidhya.com/blog/2014/10/introduction-k-nearest-neighbors-algorithm-clustering/>

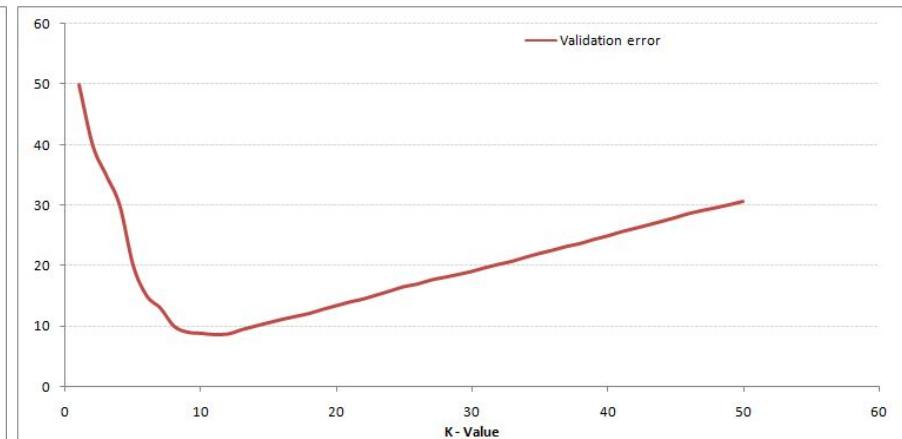
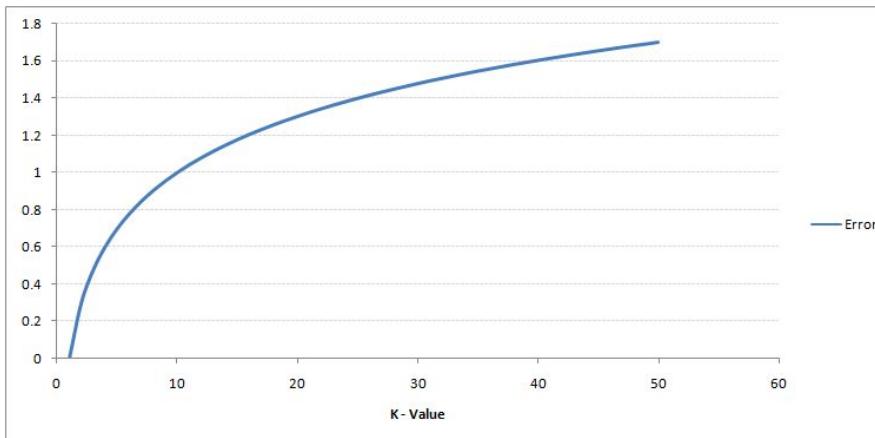
# **Question:**

## What defines a good $k$ value?



# KNN

The  $k$  value you use has a relationship to the fit of the model.



[Source](#)

# Classifier 2: Naive Bayes Classifier

**Problem.** We have  $k$  classes and want to predict which class the point  $x$  belongs to.  $x$  is a vector with  $n$  features.

- Calculate the probability of  $x$  being in each  $k$
- Predict  $x$  to be the class with the maximum probability

Assuming all  $n$  features are independent, the probability of  $x$  being in each  $k$  is

$$p(C_k) \prod_{i=1}^n p(x_i \mid C_k)$$



# Probability Distribution Used

Naive Bayes classifiers differ by how they assume the distribution of  $P(x_j|C_k)$ .

**Gaussian Naive Bayes:** likelihood of features assumed to be normally distributed

**Bernoulli Naive Bayes:** The features follow a “coin-flip” model. Two outcomes, one with probability  $p$  and one with probability  $1 - p$ .

Other lesser-known distributions include [Beta](#) and [Gamma](#).



# Classifier 3: Support Vector Machine

Powerful tool with a cool name.

Great at classifying data in high dimensional spaces. Only uses subset of data, hence memory efficient.

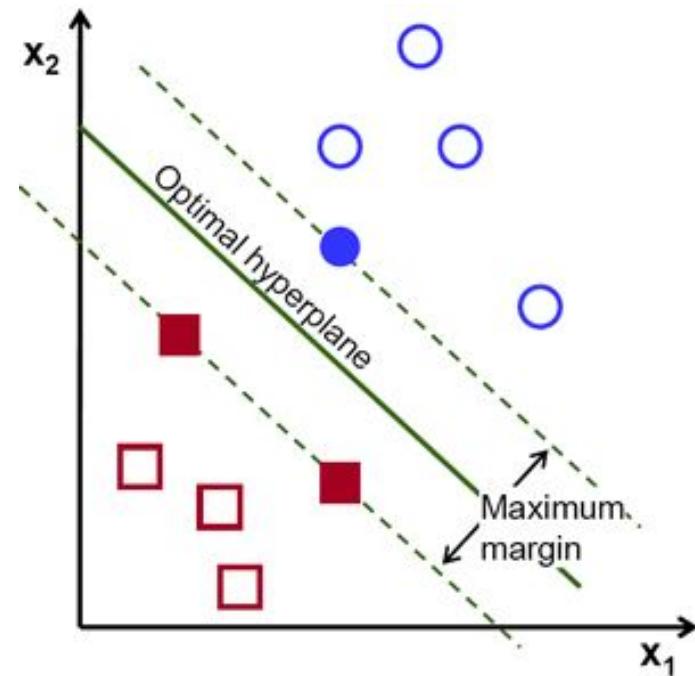
Note: requires large calculation time and doesn't handle noise well.



# Maximal Margin Classifier

We want to find a **separating hyperplane**.

Once we find some candidates for the hyperplane, we try to maximize the **margin**, the normal distance from borderline points.



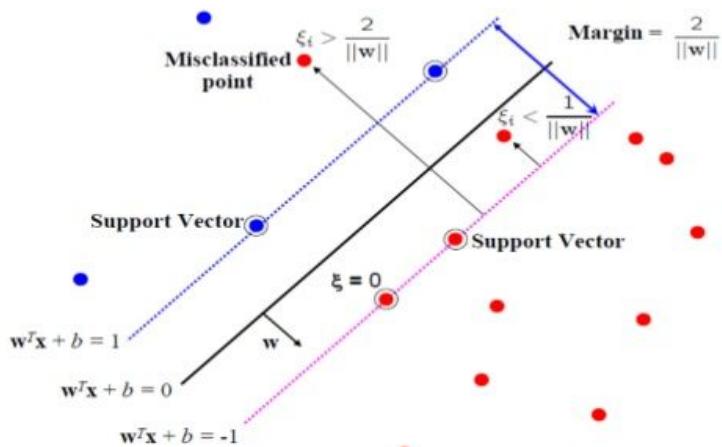
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# Hard/Soft Margins

What if the two regions are not linearly separable?

- Soft margin allows misclassification
- Can account for “dirty” boundaries

## Soft- margin SVM



$$y_i(w^T x_i + b) \geq 1 - \xi_i \quad \text{for } i = 1, \dots, M \dots\dots (7)$$

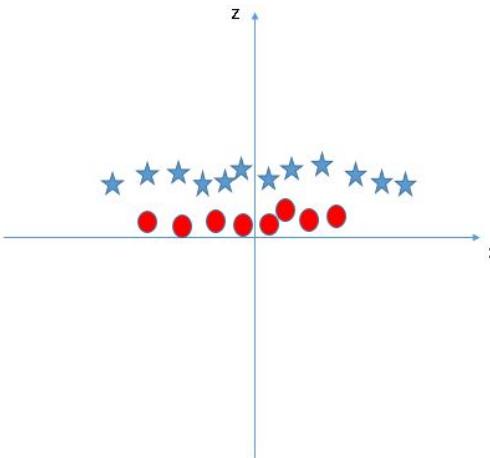
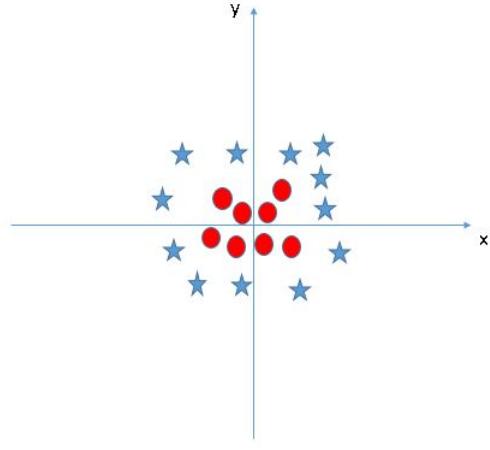


# Kernels in Action

You cannot linearly divide the 2 classes on the  $xy$  plane at right.

Introduce new feature,  $z = x^2 + y^2$  (**radial kernel**)

Map 2 dimensional data onto 3 dimensional data. Now a hyperplane is easy to find. (Imagine slicing a cone!)



[Source](#)

# Coming Up

**Your problem set:** Continue project 1

**Next week:** Clustering and unsupervised learning

